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1602

LEAVING CERTIFICATE EXAMINATION, 1998

APPLIED MATHEMATICS — ORDINARY LEVEL

FRIDAY, 26 JUNE — MORNING, 9.30 to 12.00

Six questions to be answered. All questions carry equal marks.

Mathematics Tables may be obtained from the Superintendent.

Take the value of g to be 10 m/s^2.

$\vec{i}$ and $\vec{j}$ are unit perpendicular vectors in the horizontal and vertical directions, respectively.

Marks may be lost if necessary work is not clearly shown or you do not indicate where a calculator has been used.

1. A speed limit of 20 m/s operates on a straight stretch of road [pq], where $|pq| = 1600$ m.

A motorist starts from rest at p and accelerates at 0.5 m/s^2 until reaching the speed limit. The motorist continues with a speed equal to the speed limit for a distance which is twice that travelled while accelerating. The motorist then decelerates uniformly to rest at q.

(i) Draw a speed-time graph of the motion from p to q.

(ii) Find the distance travelled at the speed equal to the speed limit.

(iii) Find the deceleration.

(iv) Find the total time for the journey from p to q.

2. A river is 30 m wide and is flowing with a velocity of 2 m/s parallel to the straight banks. The speed of a swimmer in still water is x m/s. The swimmer swims parallel to the straight banks. On reaching a distance of 100 m swimming against the current, the swimmer immediately turns and swims the same 100 m with the current. The total time for the swim is 2 minutes.

Show that the value of x is 3.

Later the swimmer swims across the river.

Find

(i) the quickest time it takes

(ii) the time it takes by the shortest path, that is, 30 m.

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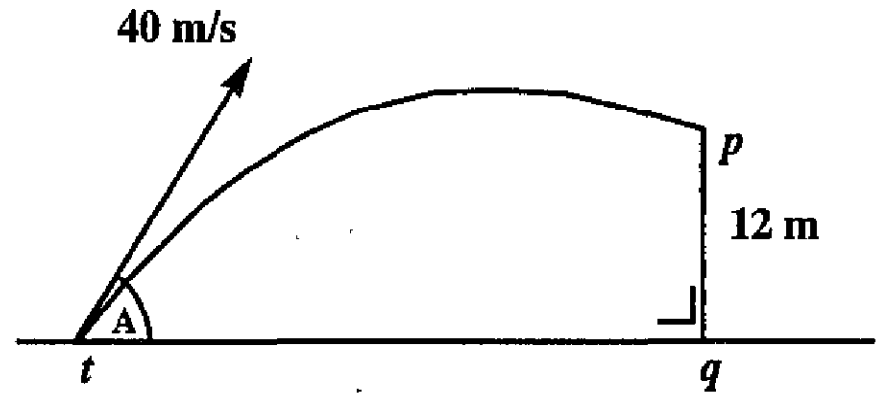
3. A golfer hits a ball from a point t on level horizontal ground with an initial speed of 40 m/s inclined at an angle A to the horizontal where $\tan A = \frac{4}{3}$.



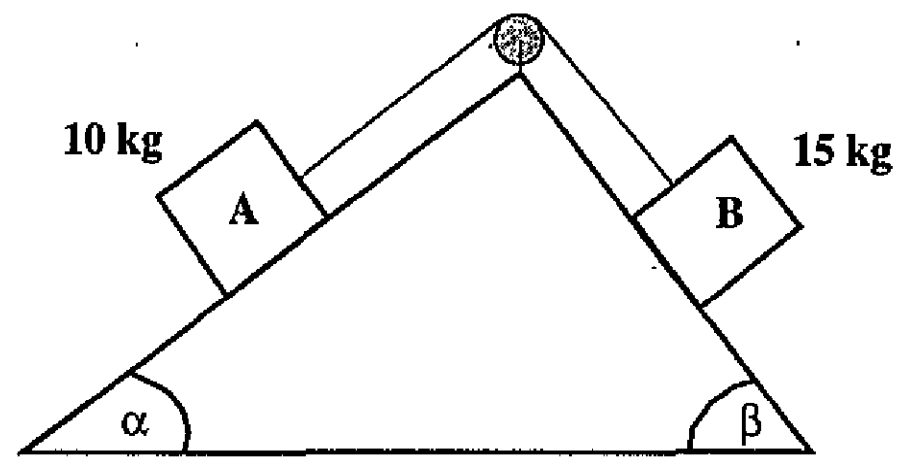
As the ball is descending it strikes a tree at a point p where $|pq| = 12$ m and pq is perpendicular to tq.

(i) Express the initial velocity in terms of $\vec{i}$ and $\vec{j}$.

(ii) Find the time taken for the ball to reach p.

(iii) Calculate $|tq|$.

4. Particles A and B, of masses 10 kg and 15 kg, respectively, are connected by a light, taut, inextensible string passing over a smooth fixed pulley. Both particles rest on rough inclined planes. Particle A lies on a plane inclined at an angle α to the horizontal where $\tan \alpha = \frac{3}{4}$ and B lies on the other plane inclined at an angle β to the horizontal where $\tan \beta = \frac{4}{3}$. The coefficient of friction between A and its plane is $\frac{1}{4}$ and between B and its plane is $\frac{1}{3}$. The particles are released from rest.

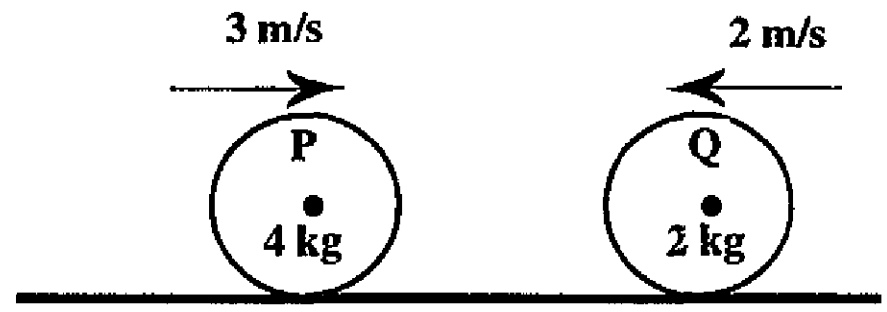


(i) Show on separate diagrams the forces acting on each particle.

(ii) Find the common acceleration of the particles.

(iii) Find the tension in the string.

5. Two smooth spheres P and Q, of masses 4 kg and 2 kg respectively, and travelling in opposite directions with speeds of 3 m/s and 2 m/s respectively, collide directly on a smooth horizontal table. The coefficient of restitution for the collision is e. As a result of the collision the change in the linear momentum of Q is $20e$ N.



(i) Show that the speed of P after the collision is

$$\frac{4 - 5e}{3} \text{ m/s.}$$

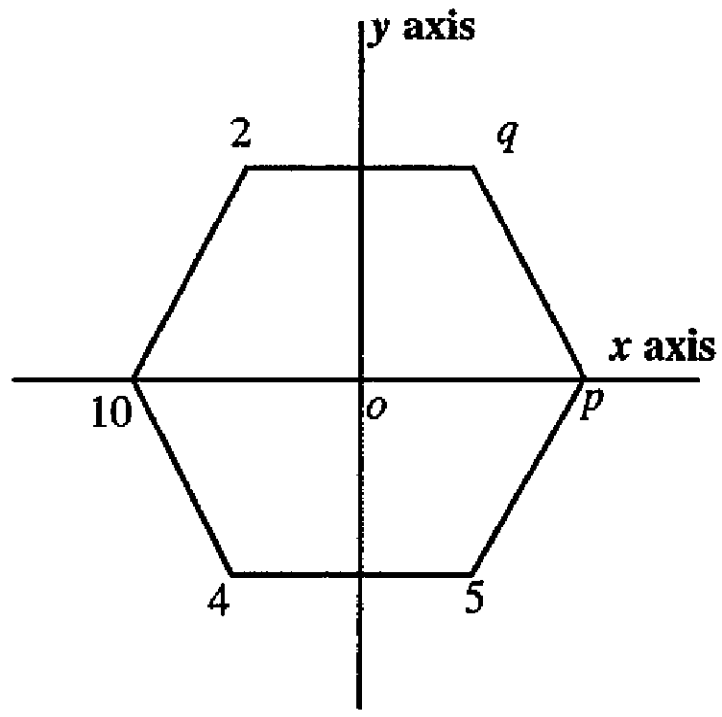
(ii) Find the value of e.

(iii) Find the speed of Q after the collision.

6. **(a)** The centre of gravity of a uniform regular hexagonal lamina, with side of length l cm, is at the origin, o.

Particles with masses, in kg, of 2, 10, 4, 5, p and q are placed at the vertices as shown.

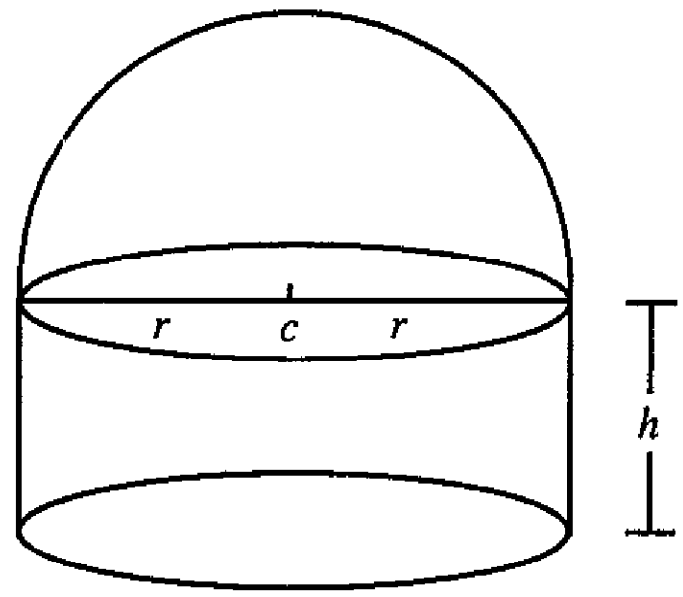
If the centre of gravity of this horizontal system is at o, find the value of p and the value of q.



(b) A uniform solid (paperweight) is in the shape of a cylinder of height h cm surmounted by a hemisphere with radius of length r cm.

If the centre of gravity of the solid is at c, the centre of the plane face of the hemisphere, as shown, find h in terms of r.

See Tables p. 40.



7. A uniform ladder, [pq], of mass 40 kg and length $2l$, has end p on rough horizontal ground and end q against a rough vertical wall. The coefficient of friction at p is $\frac{2}{5}$ and at q is $\frac{1}{2}$.

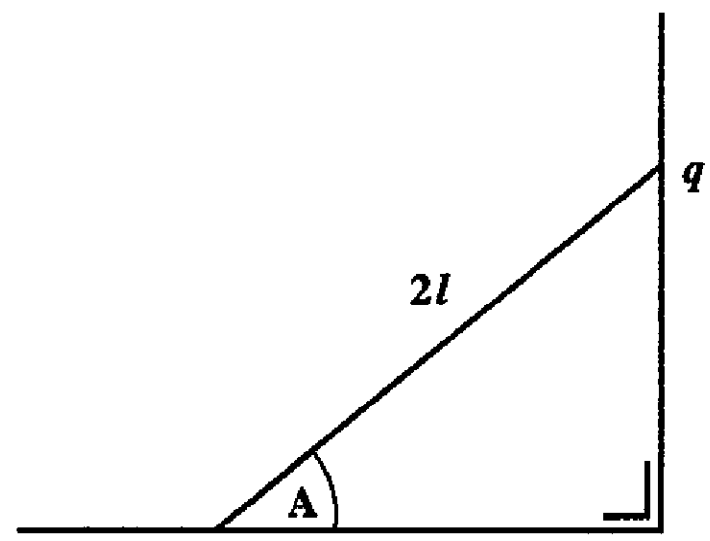
The ladder is on the point of slipping when inclined at an angle A to the horizontal.

(i) Show on a diagram all the forces acting on the ladder.

(ii) Find the values of the normal reactions at p and q.

Give your answer as a fraction in each case.

(iii) Find the value of A.



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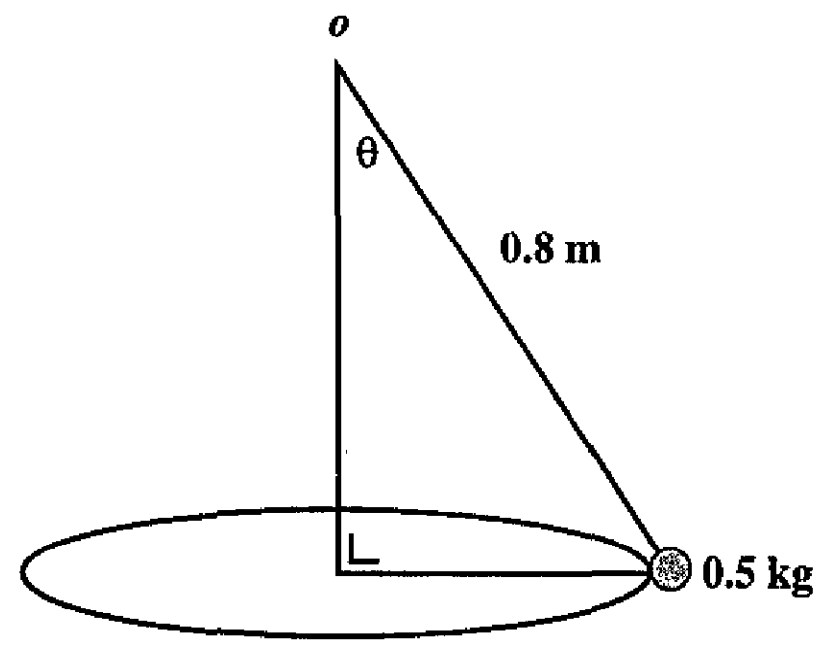
8. A particle of mass 0.5 kg is connected by means of a light, inextensible, string of length 0.8 m to a fixed point *o*.



The particle describes a horizontal circle with constant angular velocity of ω rad/s.

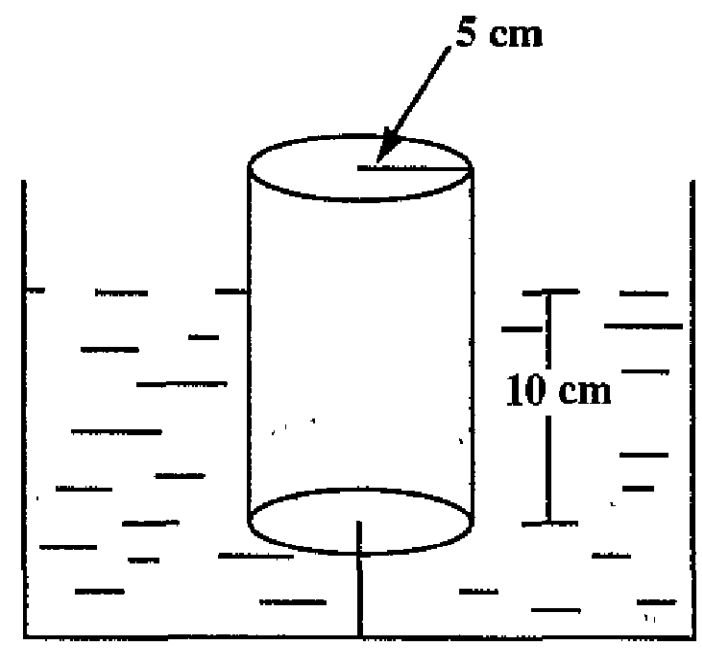
The tension in the string is 10 N.

(i) Draw a clear diagram showing all the forces acting on the particle.

(ii) Find the value of θ.

(iii) Find the angular velocity, ω, of the particle.

9. State the principle of Archimedes.



A hollow upright cylinder, with radius of length 5 cm has a mass of 0.235π kg. The upright cylinder is held partially immersed in a tank of water by a light, inextensible, string attached to the centre of its base and to the bottom of the tank. The length of cylinder immersed is 10 cm, as shown.

Show that the tension in the string, in terms of π, is 0.15π N.

Liquid of relative density 0.6 is poured into the hollow cylinder to a depth of *h* cm. The string is cut and the cylinder floats upright with the same length of 10 cm immersed as before.

Find the value of *h*.

Density of water = 1000 kg/m^3.